DO NOW

Discuss the video from yesterday with a neighbor.

Thinking big picture, what was the most significant thing you learned?

Page 1

5.2 Area

See Handout of presentation pages.

Page 2

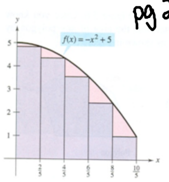
Estimate the area under the curve $f(x) = -x^2 + 5$ bounded by the y-axis and $x = 2$, using 5 rectangles of equal widths, under the curve.

pg 298

$$\text{width of each } \square = \frac{\text{length of domain}}{\# \text{ of rect.}} = \frac{2}{5}$$

$$\text{Sum of areas} = l_1\left(\tfrac{2}{5}\right) + l_2\left(\tfrac{2}{5}\right) + \ldots + l_5\left(\tfrac{2}{5}\right)$$

$$A = f\left(\tfrac{2}{5}\right)\left(\tfrac{2}{5}\right) + f\left(\tfrac{4}{5}\right)\left(\tfrac{2}{5}\right) + \ldots + f\left(\tfrac{10}{5}\right)\left(\tfrac{2}{5}\right)$$



(a) The area of the parabolic region is greater than the area of the rectangles.

$a = 0 \quad b = 2$

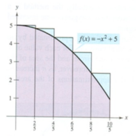
$$A = 4.84(.4) + 4.36(.4) + 3.56(.4) + 2.44(.4) + 1(.4)$$

$$A \approx 6.48$$

*Actual area will be larger

Page 3

*using rectangles above the curve, the actual area would be less.



(b) The area of the parabolic region is less than the area of the rectangles.

Using Sigma Notation:

$$\sum_{i=1}^{5} f\left(\tfrac{2}{5}i\right)\left(\tfrac{2}{5}\right) = \sum_{i=1}^{5}\left(-\left(\tfrac{2}{5}i\right)^2 + 5\right)\left(\tfrac{2}{5}\right)$$

$$\sum_{i=1}^{5}\left(-\tfrac{4}{25}i^2 + 5\right)\left(\tfrac{2}{5}\right)$$

$$\sum_{i=1}^{5} \tfrac{-8}{125}i^2 + 2$$

$$\tfrac{-8}{125}\sum_{i=1}^{5} i^2 + \sum_{i=1}^{5} 2$$

$$\tfrac{-8}{125}\left(\tfrac{5\cdot 6\cdot 11}{6}\right) + 2\cdot 5$$

$$-3.52 + 10$$

$$\boxed{6.48}$$

Page 4

The more rectangle used the better the approximation!!!!

If we let n $\rightarrow\infty$ to divide the area into an infinite number of rectangles, we will find the actual area under the curve. It wouldn't matter whether the rectangles were drawn above or below the curve!!!

Page 5

Definition of the Area of a Region in the Plane

Let f be continuous and nonnegative on the interval $[a, b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is:

length · width

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)(\Delta x)$$

where: $\Delta x = \frac{b-a}{n}$

c_i: $a + \Delta x i$

Page 6

Find the area of $f(x) = -x^2 + 5$ bounded by the x-axis on the interval [0, 2].

$$\Delta x = \frac{2-0}{n} = \frac{2}{n}$$

$$c_i = a + \Delta x i$$

$$0 + \frac{2}{n}\cdot i$$

$$c_i = \frac{2i}{n}$$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[-\left(\frac{2i}{n}\right)^2 + 5\right]\left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{-8i^2}{n^3} + \frac{10}{n}$$

$$\lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{-8i^2}{n^3} + \sum_{i=1}^{n} \frac{10}{n}\right]$$

$$\lim_{n\to\infty} \left[\frac{-8}{n^3}\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} \frac{10}{n}\right]$$

$$\lim_{n\to\infty} \left[\frac{-\cancel{8}^{4}}{\cancel{n^3}_{n^2}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{3}} + 10\right]$$

$$\lim_{n\to\infty} \left[\frac{-4\cdot(2n^2+3n+1)+30n^2}{3n^2}\right]$$

$$\lim_{n\to\infty} \left[\frac{-8n^2-12n-4+30n^2}{3n^2}\right]$$

$$\lim_{n\to\infty} \left(\frac{22n^2-12n-4}{3n^2}\right)$$

$$\frac{22}{3} = \boxed{7.\overline{3}}$$

Page 7

Find the area of : $f(x) = 1 - x^2$; [-1, 1]

$$\Delta x = \frac{1-{}^{-}1}{n} = \frac{2}{n}$$

$$c_i = a + \Delta x i$$

$$c_i = -1 + \frac{2i}{n}$$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1+\frac{2i}{n}\right)\left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(1-\left(-1+\frac{2i}{n}\right)^2\right)\left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(1-\left(1-\frac{4i}{n}+\frac{4i^2}{n}\right)\right)\left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i}{n}-\frac{4i^2}{n^2}\right)\left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} \left(\frac{4i}{n}-\frac{4i^2}{n^2}\right)$$

$$\lim_{n\to\infty} \frac{8}{n^2}\sum_{i=1}^{n} \left(i-\frac{i^2}{n}\right)$$

$$\lim_{n\to\infty} \left[\frac{8}{n^2}\sum_{i=1}^{n} i - \frac{8}{n^2}\cdot\frac{1}{n}\sum_{i=1}^{n} i^2\right]$$

continued...

Page 8

(continued)

$$\lim_{n\to\infty} \left[\frac{8}{n^2}\sum_{i=1} i - \frac{8}{n^3}\sum i^2\right]$$

$$\lim_{n\to\infty} \left[\frac{\cancel{8}^{4}}{\cancel{n^2}_{n}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}} - \frac{\cancel{8}^{4}}{\cancel{n^3}_{n^2}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{3}}\right]$$

$$\lim_{n\to\infty} \left[\frac{4n+4}{n} - \frac{4(2n^2+3n+1)}{3n^2}\right]$$

$$\lim_{n\to\infty} \left[\frac{3n(4n+4)-8n^2-12n-4}{3n^2}\right]$$

$$\lim_{n\to\infty} \left[\frac{12n^2+12n-8n^2-12n-4}{3n^2}\right]$$

$$\lim_{n\to\infty} \left(\frac{4n^2-4}{3n^2}\right)$$

$$\frac{4}{3} = \boxed{1\frac{1}{3}}$$

Page 9

HOMEWORK

pg 304 - 305; 23, 29, 47 - 53 odd

Page 10